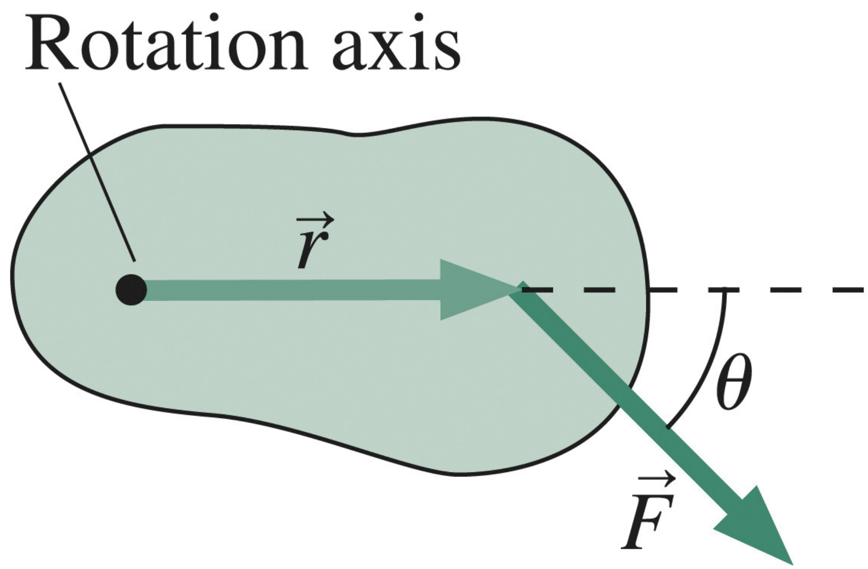
Chapter 12: Static Equilibrium
Thursday March 19th

- Review of Torque and Newton's 2nd law
- The requirements for equilibrium
- Static equilibrium
- Stable and unstable equilibrium
- Examples, demonstrations and *i*clicker

- I will be away on Tuesday – Dr. Hori to cover for me.
- My office hours today from 11-12:30; also next Thu. 11-noon.
- Tuesday's lecture jumps back to Chapter 8 on Gravity.
- Mini-Exam next Thursday 26th (LONCAPA #13-17).

Reading: up to page 195 in Ch. 12

Review: Torque and Newton's 2nd Law



Definition: $$\tau = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta$$

Newton's 2nd law: $$\tau = I\alpha$$

Rotational equivalent of force

Rotational equivalent of mass

Rotational acceleration

Equilibrium

A system of objects is said to be in equilibrium if:

1. The linear momentum $\vec{P}$ of its center of mass is constant.
2. Its angular momentum $\vec{L}$ about its center of mass, or about any other point, is also constant.

If, in addition, $\vec{L}$ and $\vec{P}$ are zero, the system is said to be in **static equilibrium**.

Examples of dynamic equilibrium:

- Ice hockey puck sliding on frictionless ice
- An object in free fall having reached terminal velocity
- A ball rolling unimpeded on a horizontal surface
- A bicycle traveling at constant velocity

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Examples of static equilibrium:

- A ladder leaning against a wall
- A static pile of rocks, sand or grain
- A house of cards
- A gymnast performing the crucifix

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Examples of non-equilibrium situations:

- An object falling in a vacuum under gravity
- A rocket during launch
- Most of the time you are on a roller coaster ride
- A ladder leaning against a wall if the contacts between the wall and the ground are frictionless

The requirements of equilibrium

(1) Translational motion of a body is governed by Newton's 2nd law:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \qquad \Rightarrow \vec{F}_{net} = 0$$

(2) Rotational motion of a body is governed by Newton's 2nd law in its angular momentum form:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} \qquad \Rightarrow \vec{\tau}_{net} = \sum_{\substack{\text{about} \\ \text{any point}}} \left(\vec{r}_i \times \vec{F}_i \right) = 0^{*}$$

1. The vector sum of all the external forces that act on a body must be zero.
2. The vector sum of all the external torques that act on a body, **measured about any axis**, must also be zero.

*Turns out that if this is true for one point, it is true for any point.

The requirements of equilibrium

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Balance of forces	Balance of torques
$\vec{F}_{net,x} = 0$	$\vec{\tau}_{net,x} = 0$
$\vec{F}_{net,y} = 0$	$\vec{\tau}_{net,y} = 0$
$\vec{F}_{net,z} = 0$	$\vec{\tau}_{net,z} = 0$

One more requirement for static equilibrium:

3. Linear & angular momenta of the system must be zero.

Stable and Unstable Equilibrium

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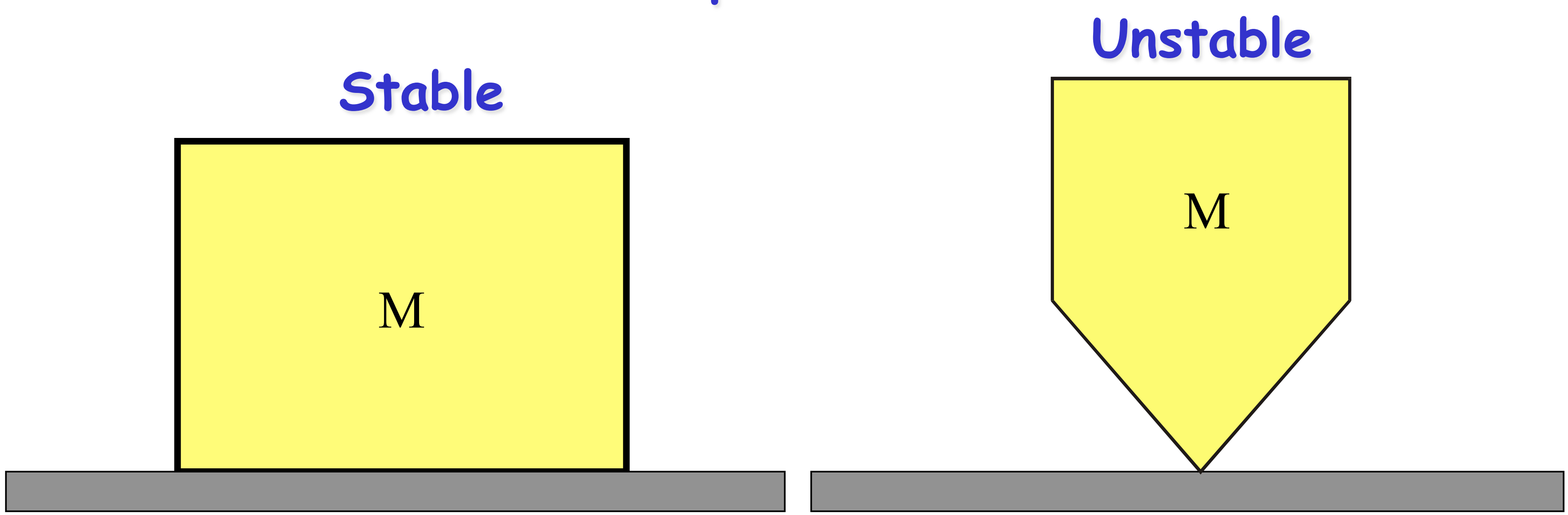
Stable versus unstable equilibrium:



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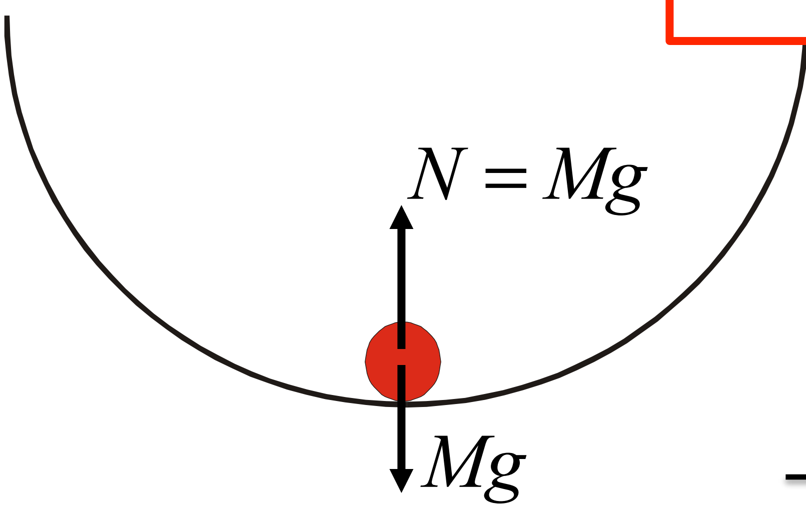
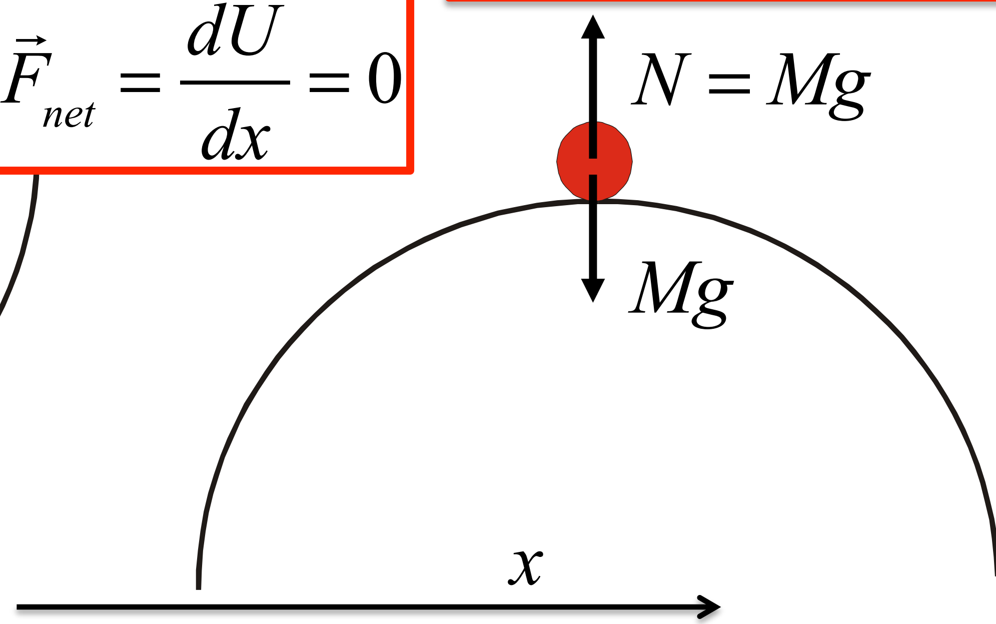
Unstable: $\frac{d^2U}{dx^2} < 0$

Stable: $\frac{d^2U}{dx^2} > 0$

$$\sum \vec{F}_{net} = \frac{dU}{dx} = 0$$





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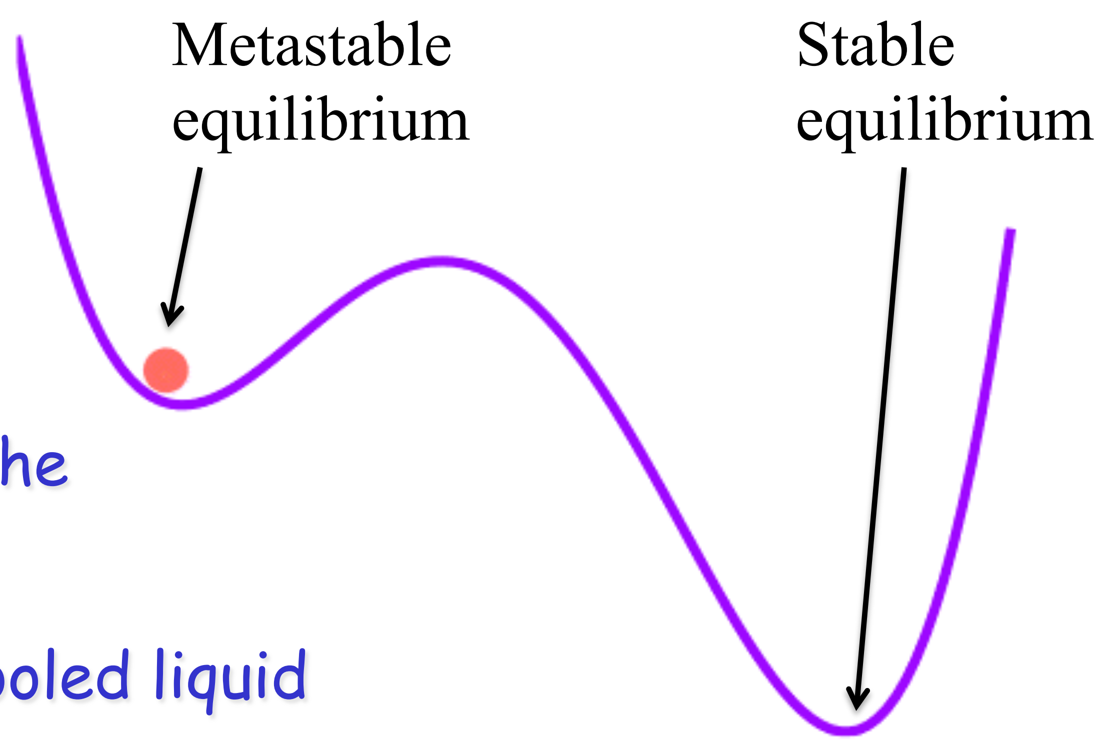

Examples:

- An avalanche
- Heat pack
- A super-cooled liquid

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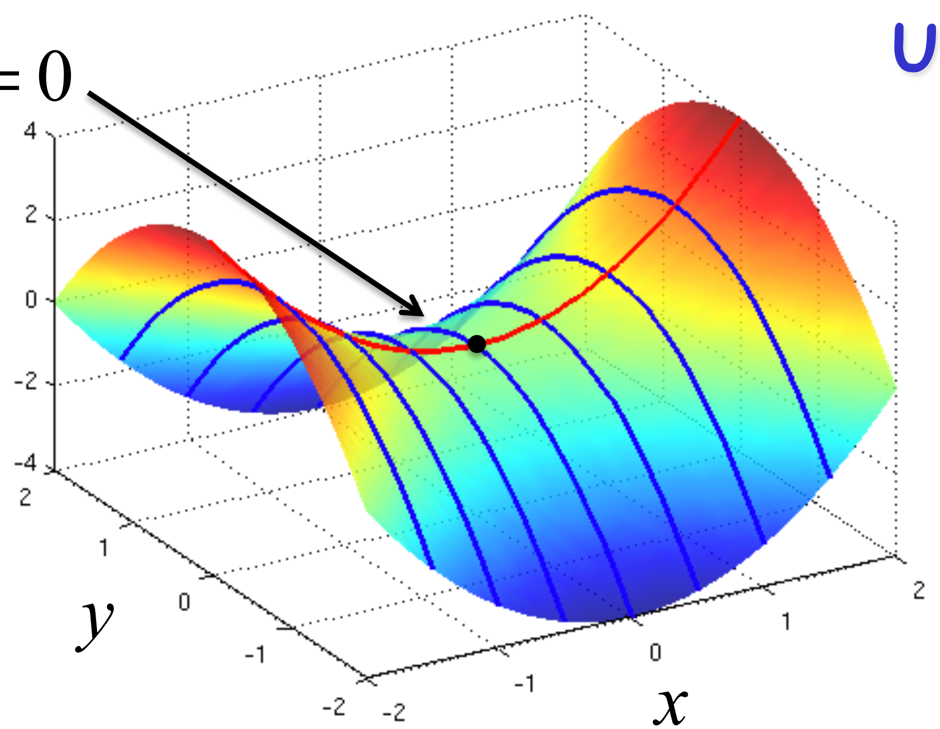
$$\frac{dU}{dx} = \frac{dU}{dy} = 0$$

Unstable



$$\frac{d^2U}{dy^2} < 0$$

$$\frac{d^2U}{dx^2} > 0$$